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AUTHOR(S): Stirling A. Colgate and Albert G. Petschek

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NEUTRINOS AND SUPERNOVA COLLAPSE

by

Stirling A. Colgate and Albert G. Petschek
New Mexico Institute of Mining and Technology
Socorro, NM 87801

and

University of California
Los Alamos Scientific Laboratory
P.O. Box 1663
Los Alamos, NM 87545

ABSTRACT

The neutrino emission resulting from stellar collapse and supernova formation is reviewed. The electron capture and consequent neutronization of the collapsing stellar matter at the end of evolution determines both the initial adiabat of core collapse as well as the trapped lepton fraction. The initial lepton fraction, $Y_0 = .48$ supplies the pressure for neutral support of the star at the Chandrasekhar limit. High trapping values, $Y_0 = .4$, lead to soft core collapses; low values to harder collapses. The value of Y_0 is presently in dispute. The neutrino emission from initial electron capture is relatively small. A strong core-bounce shock releases both electron neutrinos as well as thermal μ and τ neutrinos. Subsequent neutrino emission and cooling can sometimes lead to an unstable buoyancy gradient in the core in which case unstable core overturn is expected. Calculations have already shown the importance of the largest possible eddy or equivalently the lowest mode of overturn. Present models of low lepton trapping ratio lead to high entropy creation by the reflected shock and the stabilization of the core matter against overturn. In such cases the exterior matter must cool below an entropy of approximately $s/k \cong 2$ to become unstable. This may require too long a time, approximately one second for neutrino cooling from a neutrinosphere at $\rho \cong 2 \times 10^{12}$ g cm⁻³. On the other hand, high values of Y_0 such as .4 lead to softer bounces at lower density and values of the critical stabilizing entropy of 3 or higher. Under such circumstances, core overturn can still occur.

INTRODUCTION

Supernovae (SN) are believed to occur primarily due to the collapse of an evolved star to a neutron star. Some of the gravitational binding energy of the neutron star is transferred to the outside of the star which is ejected. The bulk of the binding energy of the neutron star is emitted as neutrinos. These neutrinos from galactic as well as extragalactic SN are one of the major possible sources of signal for Dumband. An alternate explanation of SN is that the pre-SN evolved star contains a carbon-oxygen core that burns or detonates with sufficient energy to disrupt and disperse the whole star. Such events would emit roughly 10^5 of the neutrino flux of a collapse event. The neutrino flux would be just that due to beta decay of the thermonuclear reaction products. This is a small fraction of the beta decay expected during collapse. If such events explain SN, then one must postulate that there exist more numerous "silent" events that form neutron stars without either light or mass emission because the frequency of occurrence of neutron stars equals or exceeds conservative estimates of visual SN

is insufficient to create the SN mass ejection, we expect subsequent cooling and deleptonization of the sub-neutrinosphere matter will still permit explosive core overturn.

SOFT BOUNCE

If the trapped lepton fraction is increased to $Y_l > 0.4$ then the bounce becomes much softer, lower density and closer to the conditions calculated by Livio et al. Furthermore, the stabilizing entropy increases because of the larger initial degenerate pressure. Under these circumstances we expect core overturn. There is already some speculation of a larger trapped lepton fraction because of the reduced electron capture beta decay rate in very heavy nuclei because of shell structure (Flowers, Fowler, and Newman 1980). If this turns out to be so, then convective core overturn again becomes feasible.

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events. The total neutrino emission from such a "silent" event would nevertheless be comparable to that from a "standard" collapse model SN because the neutron star binding energy is the source of the neutrinos in either case. The rate of neutrino emission is, however, sensitively dependent upon the model of collapse and explosion. Presently there is no widely accepted model that explains in a convincing fashion the mechanism of a collapse SN event. Because of this, the expected neutrino pulse is subject to large uncertainty.

There are four phenomenologically separable sources of neutrinos in "standard" SN collapse model. These are:

1. Electron capture by heavy nuclei resulting from the increasing Fermi level during the initial unstable collapse from 10^9 to 10^{11} g cm⁻³ density. Leptons are trapped during further collapse due to the large neutral current cross sections of heavy nuclei. The lepton number fraction (Y_ℓ leptons per baryon) is established by the low energy, slow ($\lesssim 1$ s), early collapse phase. Since Y_ℓ decreases in this phase from .48 to variously 0.4 to 0.3, the corresponding electron neutrino emission can be estimated.
2. The strong shock wave from the elastic bounce of the homologous core. These neutrinos are primarily thermal in origin so that all types are emitted from the shock wave, provided $\rho \lesssim 10^{12}$ g cm⁻³. Their emission cools and weakens the shock wave. The strength of this shock critically determines both mass ejection as well as neutrino flux. The strength of the shock wave depends sensitively upon details of the model.
3. Overturn (if it occurs) which will release the neutrinos trapped in the core in several milliseconds. The residual inner core has a high pressure due to the presence of degenerate leptons ($\gtrsim 100$ MeV Fermi level) and so, as the outer core emits leptons, the possibility exists that the core could overturn violently not only releasing the neutrinos trapped in the core but also turning gravitational and internal energy of the core into kinetic energy. This effect depends critically upon the entropy and hence relative buoyancy of the deleptonized matter of the outer core remaining from the core-bounce shock wave.
4. Diffusion of neutrinos out of the cooling core which requires only seconds even in the absence of violent overturn

NEUTRINO OSCILLATION AND COLLAPSE

Neutrino oscillations (Wolfenstein 1978) can affect hydrodynamic collapse of the core because they reduce the pressure of lepton trapped matter. The initial lepton number Y_ℓ of the trapped matter is established by both the rates of electron capture on heavy nuclei as well as neutrino diffusion during the initial collapse. Both processes are somewhat uncertain so that estimates of Y_ℓ (in the core) vary between 0.42 and 0.25. This large variation will in turn cause an uncertainty in core collapse and subsequent phenomena greater than that due to neutrino oscillations alone. To understand this we give estimates of the fractional pressure of the trapped matter for various assumptions of oscillations and Y_ℓ . The fractional support pressure is the ratio of pressure to the pressure that would exist if all leptons remained electrons starting with iron, i.e., $Y_\ell = Y_e = 0.48$. The latter pressure is the pressure that will just support a mass of $1.4 M_\odot$ (Chandrasekhar mass) in neutrally stable, hydrostatic equilibrium. This is the presumed starting condition of the pre-SN core. Neutral support implies

equilibrium at any radius and any average density and is possible because the relativistic degenerate adiabatic index has the value $4/3$. The fractional support pressure is both a measure of the degree of free-fall collapse as well as a measure of the fractional mass of the homologous core. We calculate this at a density less than nuclear density but high enough that the neutron proton mass difference can be neglected relative to the lepton energy. Then the fractional support pressure is:

$$(1 + 2^{1/3} n f^{4/3})(1 + n f)^{-4/3} (Y_\ell / .48)^{4/3}$$

where $Y_\ell = Y_e + \sum_i Y_{\nu i}$ and n is the number of neutrino types of assumed one

helicity. The parameter f measures the suppression of electron neutrinos due to the chemical potential of the excess neutrons. This suppression results in values of f of about 0.15 (Bethe, Applegate and Brown 1980). Table I shows the fractional support pressure for various Y_ℓ 's as well as for various assumption of neutrino types.

TABLE I

Fractional Support Pressure for $f = 0.15$, Various Values of Y_ℓ and Neutrino Oscillation Characteristics

P_ℓ / P_e	Y_ℓ	No.	ν 's
$\cong 0.91$	0.48	1	full trapping
0.46	0.29	1	60% trapping
0.79	0.48	3	Majorana mass
0.40	0.29	3	Majorana mass
0.68	0.48	6	Dirac mass

One sees that the uncertainty in Y_ℓ leads to a greater variation in the pressure defect than does increasing the number of neutrinos. Thus, in order to calculate SN collapse, we will certainly have to understand the possible neutrino types; nevertheless at the present state of uncertainty neutrino oscillations are not the dominant question. Furthermore it has been pointed out by Wolfenstein (1978) that neutrino oscillations are suppressed in matter whose density is as great as $10^{13} \text{ g cm}^{-3}$ because the index of refraction is different for electron neutrinos than for μ or τ neutrinos.

TRAPPING ENERGY

The difference in binding energy of the collapsed and initial core is available energy. Presumably a fraction of this energy produces the presently observable effects of SN and the remainder is available as neutrino emission.

Gudmundsson and Buchler (1980) have calculated this energy for various trapped lepton fractions Y_ℓ (Fig. 1). One sees that for the typical homologous core of $.75 M_\odot$, the binding energy is roughly 20 MeV per nucleon for a typical trapping value of $Y_\ell = 0.3$. This energy is available to form the core-bounce shock and the

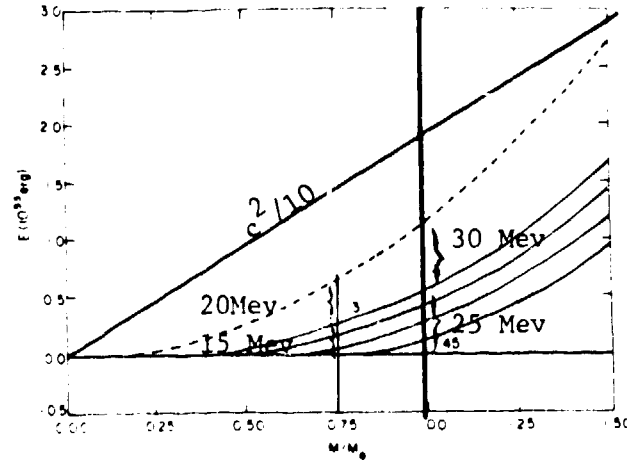


Fig. 1. The binding energy of a neutron core for various lepton trapping ratios using the Baym et al (1971) equation of state and as calculated by Gudmundsson and Buchler (1980). We have added an energy scale that emphasizes the binding energy per nucleon for equilibrium matter (cold neutron star) and a trapping ratio of $Y_l = 0.3$ for core masses of $0.75 M_\odot$ and $1.0 M_\odot$.

fast burst of neutrinos. The further binding energy available when $Y_l \rightarrow 0$ is approximately 20 MeV per nucleon also. Larger cores have larger specific energies available. If this further energy were to diffuse out of the core as neutrinos, the time required would be several seconds, long compared to hydrodynamic times of milliseconds. Therefore, diffusive release of neutrinos would not contribute to the SN mass ejection. Instead we have proposed that the residual energy of lepton trapping become observable and contributes to the SN mechanism by a process of violent unstable core overturn, (Fig. 2). (Colgate and Petschek 1980; Livio, Buchler, and Colgate 1980; Bruenn, Buchler, and Livio 1979). As the trapping fraction Y_l increases beyond .3 the fraction of energy available from overturn exceeds that available for the core-bounce shock. The question of overturn may then be critical to SN.

There are two primary questions concerning the possibility of sudden core overturn.

1. Does an unstable buoyancy gradient exist at any time, lasting long enough to permit unstable overturn?
2. Will the lowest mode, $L=2$, corresponding to overturn dominate, the convective motions above smaller scale convection despite the existence of an unstable atmosphere covering many density or pressure scale heights?

The original concept of Epstein (1979) took question 1 more or less for granted. However, a combination of high exterior entropy values ($S/k \gtrsim 3$) in some present numerical calculations from the core-bounce shock as well as from degenerate electron capture has led to serious doubts that a global unstable gradient will occur before a time of roughly a second. (Convection will start in the mantle much earlier.) By the time of one second collapse will have terminated and diffusive release of trapped leptons will have dissipated the stored energy and excess buoyancy (Mazurek and Lattimer 1980). We will go into this problem more deeply, but first we will answer the second question in the affirmative.

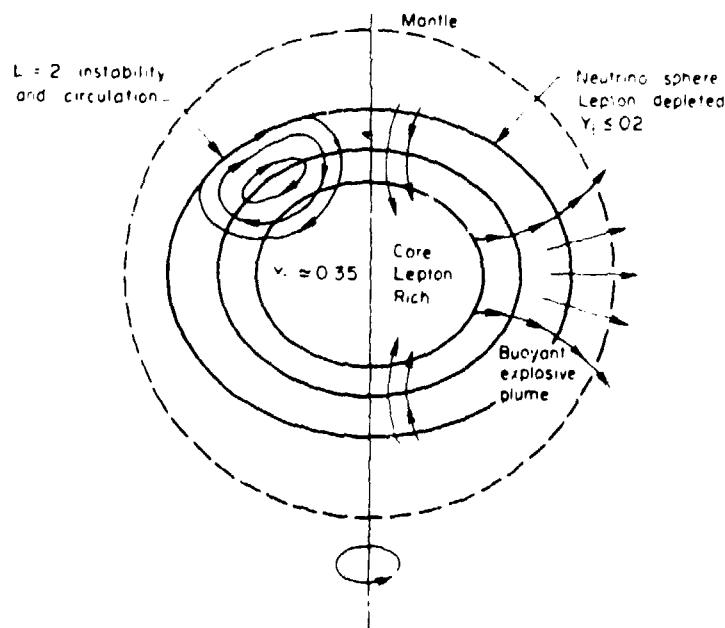


Fig. 2. Two extreme limits of the fluid flow expected from the unstable overturn of the partially de-leptonized neutron-star core during formation. The circulation shown on the left occurs if the $\ell=2$ instability grows with a relatively small unstable potential difference. The explosive plume on the right occurs if the instability potential is large, i.e., the difference in potential energy between inner and outer core is comparable to the internal energy of the de-leptonized outer core.

A numerical calculation of overturn with a global unstable gradient has been completed by Livio, Buchler, and Colgate (1980) and demonstrates as in Figs. 3 and 4 the overturn of the neutron star matter. In Fig. 4 the "swallowing" of a higher mode convective element ($L=8$) by a slower growing, but dominating $L=2$ mode is demonstrated.

These calculations were done with a different equation of state than the one currently considered most likely, and hence, the bounce occurred at lower densities ($\approx 2 \times 10^{13} \text{ g cm}^{-3}$) than in calculations using the equation of state by Bethe, Brown, Applegate and Lattimer (1979). We note parenthetically that a soft equation of state implies low pressure at a given density which delays the bounce and produces a strong outgoing shock. Thus soft equation of state implies hard bounce and vice-versa. This core-bounce shock wave produces a high entropy in the envelope that stabilizes the mantle against overturn with the core. Another two dimensional calculation of convective overturn starting from a one dimensional calculation of collapse was completed by Smarr, Wilson, Barton, and Bowers (1980). The 1-D calculations produced a strong core-bounce shock and high entropy ($s/k \gtrsim 4$) in the outer $\frac{1}{4} M_{\odot}$ of the core. Only a small fraction ($\approx \frac{1}{4} M_{\odot}$) of the distribution was unstable relative to an inner $\frac{1}{4} M_{\odot}$, which in turn was supported by a stable $\frac{3}{4} M_{\odot}$ core. The two $\frac{1}{4} M_{\odot}$ regions indeed "violently" overturned and further substantiated the previously envisaged predominance of the largest possible eddy size. Nevertheless, Smarr et al (1980) cite their calculations as conclusive evidence against overturn.

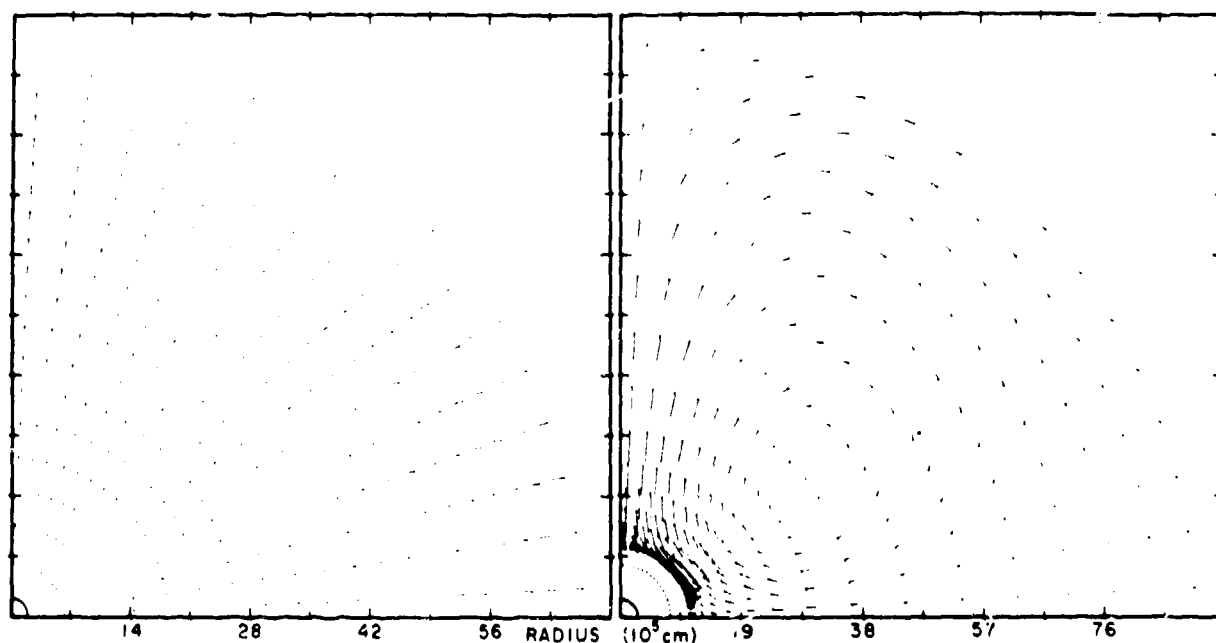


Fig. 3. (a) Velocity field for $\ell = 2$ perturbation 20 ms after the first bounce. Largest velocity of the order of 10^9 cm s^{-1} . Squares denote arrow tails. (b) Velocity field for $\ell = 2$ perturbation 31 ms after the first bounce. Largest velocity of the order of $3 \times 10^9 \text{ cm s}^{-1}$. Squares denote arrow tails.

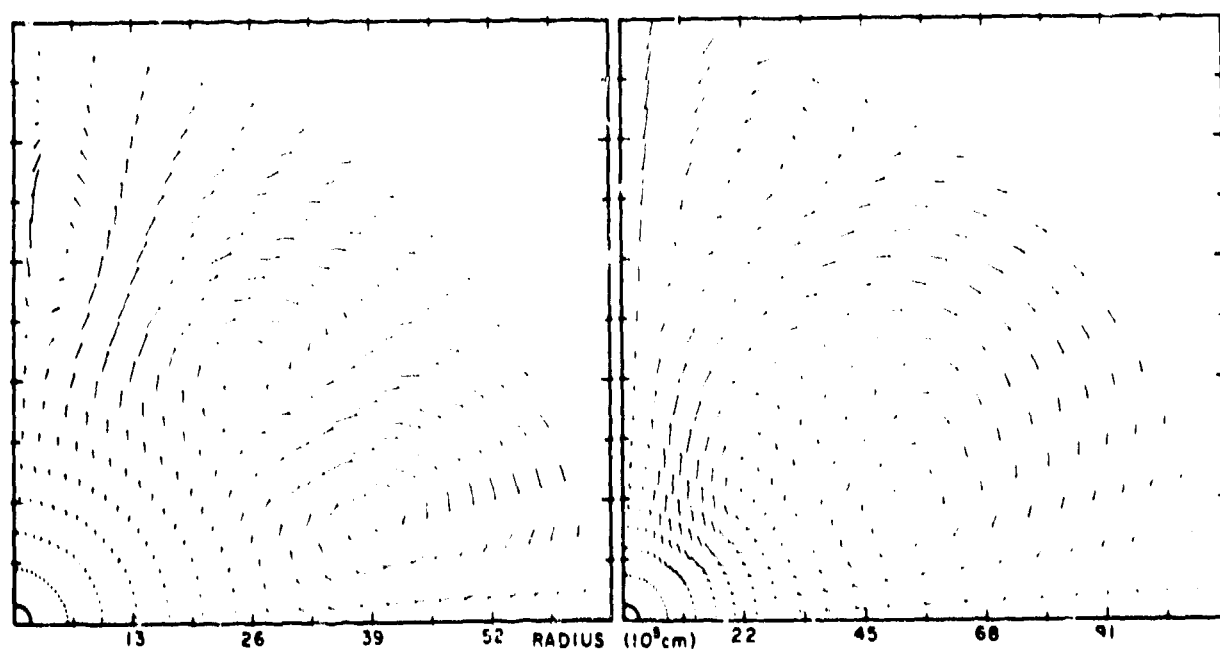


Fig. 4. (a) Velocity field for $\ell = 2$ and $\ell = 8$ perturbations 21 ms after the first bounce. Largest velocity of the order of $5 \times 10^8 \text{ cm s}^{-1}$. Squares denote arrow tails. (b) Velocity field for $\ell = 5$ perturbations 28 ms after the first bounce. Largest velocity of the order of $1.6 \times 10^9 \text{ cm s}^{-1}$. Squares denote arrow tails.

ENTROPY, LEPTON, AND STABILITY GRADIENTS

If the entropy distribution initially and after bounce remains sufficiently high on the exterior relative to the trapped low entropy interior core, then a stable buoyancy gradient may exist. That is

$$\left(\frac{1}{\rho} \frac{d\rho}{dr} - \frac{\gamma}{P} \frac{dP}{dr}\right) \text{ may be negative everywhere.} \quad (2)$$

Then, of course, the core will remain stable and not overturn. The question of the time evolution of stability is difficult to answer.

The original objection to core overturn based upon stabilizing entropy gradients was proposed by Mazurek and Lattimer (1980). They showed that a core of density around $10^{14} \text{ g cm}^{-3}$, a trapped value of $Y_\ell = .3$ and low core entropy $S/k = 1.2$ was stable against interchange with matter at larger radius, lower density, depleted in leptons to $Y_\ell < 0.1$ and having relatively modest entropies ($s/k \gtrsim 2$). Let us explain further what this means. The instability in question presumes that cold lepton trapped matter has a higher pressure at a given density (or conversely lower density at a given pressure) than external matter which has emitted its leptons by electron capture and neutrino escape. This difference in buoyancy is due to the presence or absence of the degenerate pressure of the trapped leptons. However, as Mazurek and Lattimer (1980) pointed out, one can substitute thermal pressure for degenerate lepton pressure in the lepton depleted matter. The question is how much entropy or thermal pressure is required to substitute for the degeneracy pressure?

We assume that the interchange or "overturn" is adiabatic and hence entropy is conserved in the process. Indeed condition (2) is based on just such an adiabatic interchange of two fluid elements. Hence, the question of entropy stabilization must start with an analysis of the magnitude of the entropy required to stabilize the interchange of dense, cold, lepton trapped matter and hot, lepton depleted matter.

We have completed such an analysis from B²AL for this conference, with particular emphasis upon the high density region $\rho \geq \rho_{\text{nuc}} = 2.7 \times 10^{14} \text{ g cm}^{-3}$. Unfortunately, the initial treatment of this problem (Mazurek and Lattimer 1980) was for densities $< 10^{14} \text{ g cm}^{-3}$, which therefore did not include the core region where the gravitational potential is largest and where the most energy can be generated by an interchange. The strong core-bounce shock and resulting high entropy in the exterior matter occurs when the central density is \gtrsim nuclear density because only then is the pressure increase great enough to reverse the collapse abruptly. Our initial analysis is pessimistic, in that it requires little entropy to stabilize a partially lepton trapped core against overturn.

We ask how much entropy is required to make up for the pressure defect caused by reducing the lepton number at a given density. Then this entropy is the critical stabilizing entropy. We assume the initial entropy is not zero (cold) but finite ($s/k = 1.2$) corresponding to values calculated for initial collapse models (B²AL). We similarly assume that the depleted lepton matter does not correspond to $Y_\ell = 0$, but instead is only partially depleted ($Y_\ell = 0.1$). In this fashion, we expect some nuclear binding to exist in the neutron rich matter. Then following

B^2_{AL} , the nuclear pressure is proportional to $T^2 \propto s^2$ and the pressure at entropy s_2 becomes

$$P(s_2) = P(s_1) \frac{s_2^2}{s_1^2}. \quad (3)$$

Using this and the figures given in their tables 5 and 6, we estimate the values of critically stabilizing entropy s/k in our Table II. In this analysis, we have neglected the relatively minor contribution to the pressure of the heat added to the leptons. The slight variation of s/k with ρ is due to deviations of the equation of state from a perfect gas.

TABLE II

Critical Stabilizing Entropy for Material with $Y_2 = 0.1$
with Respect to Core Material of $Y_2 = 0.3$,
 $s/k = 1.2$ and density ρ_0

ρ_0/ρ_{nuc}	s/k
1	2.25
1.5	2.0
2	2.0

We see that $s/k \approx 2$ is sufficient to stabilize the dense core against overturn for initial $Y_2 \leq 0.3$.

Mazurek and Lattimer (1980) predict that at lower densities significantly higher entropies are required for stabilization (i.e. $s/k = 3$ to 4). A strong shock will give still greater entropies ($s/k = 7$ to 8). Thus we see that the most likely remaining circumstance for convective core overturn is that conditions result in a weak shock similar to those already calculated by Livio et al (1980).

The core-bounce shock deposits an entropy of $s/k \approx 10$ at the neutrino sphere at density $= 10^{12} \text{ gm}^{-3}$ and $T \approx 10 \text{ Mev}$ (Van Riper 1980). We would interpret this high entropy as more than sufficient to stabilize against overturn with core lepton-trapped matter. Here we have assumed that the core-bounce shock, although very strong is still insufficient to eject the supernova mantle. Then we must consider cooling. The low density matter ($\rho \leq 10^{12}$) cools slowly to $\approx 2 \text{ Mev}$ where $s/k \leq 2$ because electron pair neutrino emission is weak ($\propto T^9$) and nuclear excited state emission (Kolb and Mazurek 1979) is inhibited because nuclei are thermally decomposed to free nucleons. Instead we consider matter at higher density ($\rho \approx 2 \times 10^{13} \text{ g cm}^{-3}$) where an entropy of $s/k = 2$ correspond to $T = 6 \text{ Mev}$. Then the cooling time is very short - several milliseconds and neutrino diffusion to the neutrinosphere surface governs the time constant. This time constant very roughly calculated is less than a few 10's of milliseconds. If thermal neutrinos are emitted fast enough to cool the matter to an entropy that permits overturn, then the excess electron neutrinos can similarly escape. Thus we see an unstable gradient developing much as we envisaged in Colgate and Petschek (1980). Then if a relatively strong core-bounce shock derived from a high central density bounce

is insufficient to create the SN mass ejection, we expect subsequent cooling and deleptonization of the sub-neutrinosphere matter will still permit explosive core overturn.

SOFT BOUNCE

If the trapped lepton fraction is increased to $Y_0 > 0.4$ then the bounce becomes much softer, lower density and closer to the conditions calculated by Livio et al. Furthermore, the stabilizing entropy increases because of the larger initial degenerate pressure. Under these circumstances we expect core overturn. There is already some speculation of a larger trapped lepton fraction because of the reduced electron capture beta decay rate in very heavy nuclei because of shell structure (Flowers, Fowler, and Newman 1980). If this turns out to be so, then convective core overturn again becomes feasible.

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